A New Frequency-Domain TLM Symmetrical Condensed Node Derived Directly from Maxwell's Equations

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Abstract --- This paper presents a new frequency-domain TLM symmetrical condensed node derived directly from Maxwell's equations by using centered differencing and averaging. Direct correspondence between the FDTLM and finite difference method is established. The node scattering matrices and field expressions are given for the general case with graded mesh and anisotropic materials including both electric and magnetic losses. It is demonstrated that this new FDTLM node always has 2nd-order accuracy regardless of a uniform or graded mesh discretization of the space.

1. Introduction

The frequency-domain TLM (FDTLM) method is a new numerical technique for solving 3-D electromagnetic field problems. Since its concept was first introduced by Jin and Vahldieck in 1992[1]~[3], the FDTLM method has been successfully applied to analyze a variety of 3-D electromagnetic structures[1]~[7]. The basic idea of the FDTLM is to model the original electromagnetic structure with an equivalent electric network by filling the space to be analyzed with a transmission line matrix, or node array. Electric and magnetic fields in the original electromagnetic problem are represented by voltages and currents in the network which are then treated with numerous standard network techniques. Using different nodes for different media, the FDTLM method can be applied to virtually any structure with complex geometry.

The nodes used in the FDTLM method were originally derived from the time-domain TLM (TDTLM) nodes [1]~[5]. However, it has also been shown that the FDTLM nodes can be derived directly in the frequency domain by establishing a relationship between the impedances and propagation constants of the main lines of the node and the properties of the medium to be modelled [6][7]. In this paper, we will demonstrate that the FDTLM node can be derived directly from Maxwell's equations without resorting to any circuit concept. A new symmetric FDTLM node is derived from Maxwell's equations using centered differencing and averaging. The scattering matrix of this new node is given for the general case with a graded mesh and anisotropic materials including both electric and magnetic losses. The accuracy of the TLM algorithm is also discussed.

2. Theory

In the rectangular coordinate system, Maxwell's equations in the frequency-domain can be written as:

$$j\omega\varepsilon_0\varepsilon_x \cdot e_x = \frac{\partial h_z}{\partial y} - \frac{\partial h_y}{\partial z} - \sigma_{ex}e_x \tag{1a}$$

$$j\omega\varepsilon_0\varepsilon_y \cdot e_y = \frac{\partial h_x}{\partial z} - \frac{\partial h_z}{\partial x} - \sigma_{ey}e_y \tag{1b}$$

$$j\omega\varepsilon_0\varepsilon_z \cdot e_z = \frac{\partial h_y}{\partial x} - \frac{\partial h_x}{\partial y} - \sigma_{ez}e_z \tag{1c}$$

$$j\omega\mu_0\mu_x \cdot h_x = \frac{\partial e_y}{\partial z} - \frac{\partial e_z}{\partial y} - \sigma_{mx}h_x \tag{1d}$$

$$j\omega\mu_0\mu_y \cdot h_y = \frac{\partial e_z}{\partial x} - \frac{\partial e_x}{\partial z} - \sigma_{my}h_y \tag{1e}$$

$$j\omega\mu_0\mu_z \cdot h_z = \frac{\partial e_x}{\partial y} - \frac{\partial e_y}{\partial x} - \sigma_{mz}h_z \tag{1f}$$

where ω is the frequency; ε and μ are the permittivity and permeability; σe and σm are the equivalent electric and magnetic conductivities, respectively.

The grid points in the discretized space are denoted by indices (i,j,k), respectively, in the x, y, z directions (Fig.1). Any function of space at space point (i,j,k) is expressed as $F(i, j, k)$. The cell sizes are denoted by u, v, w, respectively, along the x, y, z directions.

Equ.(1) can be simplified by using the following transformation:

$$
\begin{aligned}
&x = u \cdot X; \quad y = v \cdot Y; \quad z = w \cdot Z \\
&e_x = E_x/u; \quad e_y = E_y/v; \quad e_z = E_z/w \\
&h_x = H_x/uZ_0; \; h_y = H_y/vZ_0; \; h_z = H_z/wZ_0
\end{aligned}
\qquad (2)
$$

where Z_0 is the characteristic impedance of free space. Equ.(1) is then rewritten as:

$$0 = \frac{\partial H_z}{\partial Y} - \frac{\partial H_y}{\partial Z} - G_{ex} \cdot E_x \qquad (3a)$$

$$0 = \frac{\partial H_x}{\partial Z} - \frac{\partial H_z}{\partial X} - G_{ey} \cdot E_y \qquad (3b)$$

$$0 = \frac{\partial H_y}{\partial X} - \frac{\partial H_x}{\partial Y} - G_{ez} \cdot E_z \qquad (3c)$$

$$0 = \frac{\partial E_y}{\partial Z} - \frac{\partial E_z}{\partial Y} - G_{mx} \cdot H_x \qquad (3d)$$

$$0 = \frac{\partial E_z}{\partial X} - \frac{\partial E_x}{\partial Z} - G_{my} \cdot H_y \qquad (3e)$$

$$0 = \frac{\partial E_x}{\partial Y} - \frac{\partial E_y}{\partial X} - G_{mz} \cdot H_z \qquad (3f)$$

where:

$$
\begin{aligned}
G_{ex} &= (\sigma_{ex} \cdot Z_0 + j\varepsilon_x \cdot k_0) \cdot \frac{v \cdot w}{u} \\
G_{ey} &= (\sigma_{ey} \cdot Z_0 + j\varepsilon_y \cdot k_0) \cdot \frac{u \cdot w}{v} \\
G_{ez} &= (\sigma_{ez} \cdot Z_0 + j\varepsilon_z \cdot k_0) \cdot \frac{u \cdot v}{w} \\
G_{mx} &= (\sigma_{mx}/Z_0 + j\mu_x \cdot k_0) \cdot \frac{v \cdot w}{u} \\
G_{my} &= (\sigma_{my}/Z_0 + j\mu_y \cdot k_0) \cdot \frac{u \cdot w}{v} \\
G_{mz} &= (\sigma_{mz}/Z_0 + j\mu_z \cdot k_0) \cdot \frac{u \cdot v}{w}
\end{aligned}
\qquad (4)
$$

with k_0 denoting the wave propagation constant in free space.

A set of finite difference equations can be obtained from equ.(3) by using centered-differencing at point (i,j,k). For example, for (3a), we have:

$$
\begin{aligned}
&\left[H_z\left(i, j+\tfrac{1}{2}, k\right) - H_z\left(i, j-\tfrac{1}{2}, k\right)\right] - \\
&\left[H_y\left(i, j, k+\tfrac{1}{2}\right) - H_y\left(i, j, k-\tfrac{1}{2}\right)\right] - G_{ex} \cdot E_x(i, j, k) = 0
\end{aligned}
\qquad (5)
$$

The finite difference equations corresponding to (3b)~(3g), respectively, can be similarly constructed.

Then we replace the grid cell (Fig.1) with its equivalent transmission line network node with 12 branches (Fig.2), and transform the normalized field variables at the cell boundaries into the incident and reflected waves at the ports of the transmission lines. For example, for port one we have:

$$E_x\left(i, j-\tfrac{1}{2}, k\right) \mp H_z\left(i, j-\tfrac{1}{2}, k\right) = 2 \cdot V_1^{i,r} \qquad (6)$$

where the upper and low signs, respectively, correspond to the incident waves V^i and reflected waves V^r on the right hand side of the equations. Equ.(6) establishes a direct correspondence between the mathematical model of the finite difference method and the physical model of the frequency-domain TLM method.

By substituting (6) into (5), it is readily shown that equ. (5) can be reduced to:

$$
\begin{aligned}
&V_{12}^r + V_1^r + V_9^r + V_2^r + G_{ex} \cdot E_x(i, j, k) \\
&-V_1^i - V_{12}^i - V_2^i - V_9^i = 0
\end{aligned}
\qquad (7)
$$

The next step in the derivation is to express the field variables at the center (i,j,k) in terms of the incident voltages. This is accomplished by averaging the mixed electric and magnetic field components at point (i,j,k) along appropriate coordinate axes. The procedure is as follows. There are a total of twelve different mixed electric and magnetic field components, namely, Ex-Hz, Ex+Hz, Ex-Hy, Ex+Hy, Ey-Hx, Ey+Hx, Ey-Hz, Ey+Hz, Ez-Hx, Ez+Hx, Ez-Hy, and Ez+Hy. For each of them, we pick up its associated coordinate, which is the coordinate orthogonal to both the electric and magnetic field in the mixed field component. The mixed field component is then averaged at point (i,j,k) with respect to its associated coordinate. For example, for (Ey+Hz), its associated coordinate is x (x-axis is orthogonal to both Ey and Hz). The centered average at point (i,j,k) for (Ey+Hz) with respect to x is then calculated, and the resulting equation is as follows:

$$V_{11}^r + V_3^i - (E_y(i, j, k) + H_z(i, j, k)) = 0 \qquad (8)$$

In a similar way, the equations for the other mixed field components can be obtained. By combining equs.(7) and

(8), one can easily find the expressions of the field components at the point (i,j,k) in terms of the incident voltages:

$$E_x(i,j,k) = \frac{2\left(V_1^i + V_2^i + V_9^i + V_{12}^i\right)}{(G_{ex}+4)} \quad (9a)$$

$$E_y(i,j,k) = \frac{2\left(V_3^i + V_4^i + V_6^i + V_{11}^i\right)}{(G_{ey}+4)} \quad (9b)$$

$$E_z(i,j,k) = \frac{2\left(V_5^i + V_6^i + V_7^i + V_{10}^i\right)}{(G_{ez}+4)} \quad (9c)$$

$$H_x(i,j,k) = \frac{2\left(-V_4^i + V_5^i - V_7^i + V_8^i\right)}{(G_{mx}+4)} \quad (9d)$$

$$H_y(i,j,k) = \frac{2\left(V_2^i - V_6^i - V_9^i + V_{10}^i\right)}{(G_{my}+4)} \quad (9e)$$

$$H_z(i,j,k) = \frac{2\left(-V_1^i + V_3^i - V_{11}^i + V_{12}^i\right)}{(G_{mz}+4)} \quad (9f)$$

Equ.(9) is derived directly from Maxwell's equations using centered differencing and averaging. The scattering matrix is then obtained by simply substituting equ.(9) into equ.(8). The 12X12 scattering matrix is shown in Fig.3. The matrix elements are given as follows:

$$\begin{aligned} a &= -\frac{G_{e\alpha}}{2(4+G_{e\alpha})} + \frac{G_{m\beta}}{2(4+G_{m\beta})} \\ b &= \frac{2}{4+G_{e\alpha}} \\ c &= -\frac{G_{e\alpha}}{2(4+G_{e\alpha})} - \frac{G_{m\beta}}{2(4+G_{m\beta})} \\ d &= \frac{2}{4+G_{m\beta}} \end{aligned} \quad (10)$$

where subscripts α and β depend on where the element is located in the matrix, as illustrated in Fig.3. For example, for S29, which is given by c, subscripts for Ge and Gm are then x and y, respectively. So we have:

$$S_{29} = c = -\frac{G_{ex}}{2(4+G_{ex})} - \frac{G_{my}}{2(4+G_{my})}$$

5. Discussions and Conclusions

Historically, the TLM method, which is based on Huygen's principle, and the finite difference method, which is based on Maxwell's equations, are distinguished as two different methods, assuming Huygen's principle and Maxwell's equations represent two distinct modelling philosophies. However, this assumption is not justified since both Huygen's principle and Maxwell's equations describe the same phenomenon in nature despite the use of different 'languages'. Indeed, they are fully equivalent. We have demonstrated [8] that the time-domain TLM (TDTLM) method is a unique time-domain finite difference scheme and can be directly derived from Maxwell's equation using centered differencing and averaging. In this paper, we demonstrated that we can draw the same conclusion for the frequency-domain TLM (FDTLM) method. Therefore, the TLM method, both in time- and frequency-domain, can be considered as either a physical model of the transmission line network (Huygen's principle) or a mathematical model of the finite difference method (Maxwell's equations), depending on which concept one feels more comfortable with.

Based on the derivation presented in this paper, one can easily assess the accuracy of the FDTLM method. The errors introduced during the derivation of the field expressions (11) and the node scattering matrix (Fig.3) are only associated with the centered differencing (Equs.(7)) and averaging (Equs.(8)). Both of them have 2nd-order accuracy. It should be mentioned that the 2nd-order accuracy of the FDTLM method is still preserved in the case of a graded mesh. This is because the errors introduced during the derivation are solely depended on the properties of the node and errors for different nodes are independent of one another. The error for one node does not change when its surrounding environment is changed.

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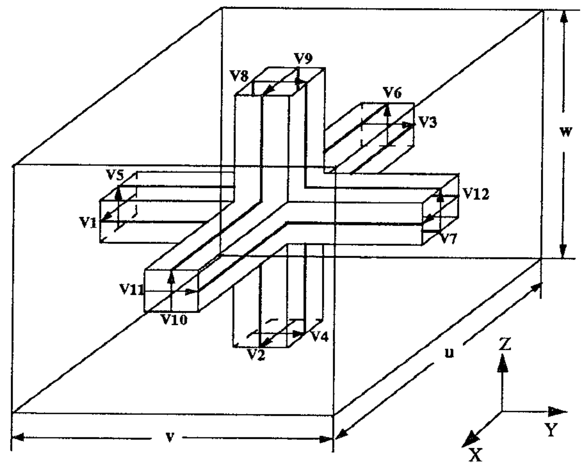

Fig.2 The equivalent FDTLM symmetrical condensed node

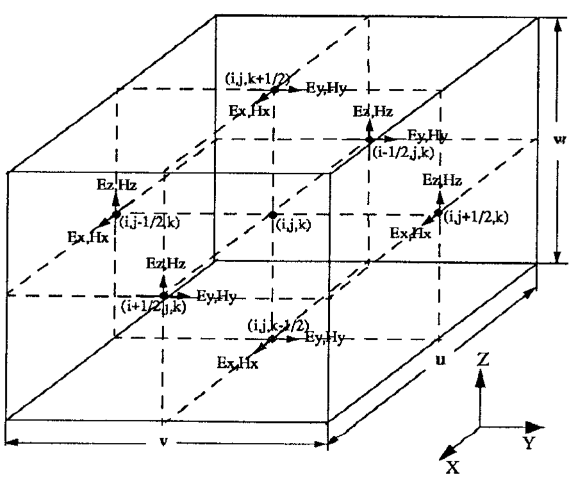

Fig.1 A unit cell and field sampling points

Column No.	1	2	3	4	5	6	7	8	9	10	11	12
Subs. α for Ge	x	x	y	y	z	z	z	y	x	z	y	x
Subs. β for Gm	z	y	z	x	x	y	x	x	y	y	z	z
1	a	b	d						b		-d	c
2	b	a				d			c	-d		b
3	d		a	b				b			c	-d
4			b	a	d		-d	c			b	
5				d	a	b	c	-d		b		
6		d			b	a	b		-d	c		
7				-d	c	b	a	d		b		
8			b	c	-d		d	a			b	
9	b	c				-d			a	d		b
10		-d			b	c	b		d	a		
11	-d		c	b				b			a	d
12	c	b	-d						b		d	a

Fig.3 The symmetrical scattering matrix of the FDTLM symmetrical condensed node